Eur. Phys. J. C 36, 453–458 (2004)
Digital Object Identifier (DOI) 10.1140/epjc/s2004-01963-5

THE EUROPEAN PHYSICAL JOURNAL C

A note on the factorization scale dependence of the PQCD predictions for exclusive processes

B. Melić[1,2,a], B. Nižić[3], K. Passek[3]

[1] Institut für Physik, Universität Mainz, 55099 Mainz, Germany
[2] Institut für Theoretische Physik, Universität Würzburg, 97074 Würzburg, Germany
[3] Theoretical Physics Division, Rudjer Bošković Institute, P.O. Box 180, 10002 Zagreb, Croatia

Received: 26 April 2004 /
Published online: 23 July 2004 –

Abstract. We briefly review the calculational procedure for the PQCD prediction for hard exclusive quantities and reconsider the problem of the factorization scale dependence.

1 Introduction

The application of perturbative QCD (PQCD) to exclusive processes at large momentum transfer is based on the factorization theorems [1–4]. The main idea is the separation of short- from long-distance effects in the sense that the high-energy region, being highly off-shell, is factorized from the low-energy region, which is characteristic of the bound-state formation. The factorization may be carried out order by order in perturbation theory. The information concerning the long-distance dynamics is accumulated in the distribution amplitude (DA), one for each hadron involved, whereas the short-distance dynamics is represented by the hard-scattering amplitude. The separation of the short- from the long-distance part occurs at the factorization scale which is usually chosen by convenience. Furthermore, PQCD calculation to the finite order necessarily requires the renormalization of the UV divergences and introduces therefore a renormalization scale dependence in the final result.

As is well known, one of the most critical problems in making reliable PQCD predictions for exclusive processes at large momentum transfer is how to deal with the dependence of the corresponding truncated perturbation series on the choice of the scheme for the QCD running coupling constant $\alpha_{\rm S}(\mu_{\rm R}^2)$ and on the choice of the renormalization scale $\mu_{\rm R}$, as well as, the factorization scale $\mu_{\rm F}$. Although the physical quantities depend neither on the renormalization nor on the factorization scale, the PQCD prediction at finite order bears the residual dependence on the renormalization and factorization scales, the choice of which introduces theoretical uncertainties in the prediction.

A lot of work has been devoted to the analysis of the renormalization scale and scheme dependence [5–8]. The problem of finding the optimal renormalization scale in a given scheme has been widely discussed in the literature and, apart from the pragmatical choice of taking $\mu_{\rm R}^2$ to equal the characteristic scale of the process, three quite different approaches have been proposed: the principle of fastest apparent convergence (FAC) [5], the principle of minimal sensitivity (PMS) [6] and the Brodsky–Lepage–Mackenzie (BLM) scale setting [7]. A physically motivated formalism in which any two perturbatively calculable observables can be related to each other without any renormalization scale or scheme ambiguity has been developed [8].

In contrast to the renormalization scale, somewhat less attention has been paid to the role played by the factorization scale. Although one can encounter in the literature several extensions of the treatments of the renormalization scale to the treatments of the factorization scale [9], when examining the hard exclusive processes the convenient choice of $\mu_{\rm F}^2$ equal to the characteristic scale of the process, i.e., the large momentum transfer denoted by, say, Q^2, is mainly used, with the justification that for such a choice the $\ln(Q^2/\mu_{\rm F}^2)$ logarithms, giving rise to the growth of the coefficients in the expansion of the hard-scattering amplitude when $Q^2 \gg \mu_{\rm F}^2$, vanish. Obviously, the result will differ for some other choice of $\mu_{\rm F}^2$. Then one can try, similarly to the renormalization scale problem, to justify other choices for the factorization scale by examining the underlying dynamics of the process [10,11].

In this paper we review the prescription and the ingredients of the higher-order calculation of the hard exclusive quantities (obtained in the so called standard hard-scattering approach [1–4]) and we reexamine their factorization scale dependence.

We show that the residual factorization scale dependence in the finite order of PQCD calculation reflects the failure of the proper resummation of all $\ln(\mu_{\rm F}^2)$ logarithms. Thus, taking into account the factorization scale depen-

[a] Alexander von Humboldt Fellow. On leave of absence from the Rudjer Bošković Institute, Zagreb, Croatia.

dence of the hard-scattering amplitude and of the distribution amplitude by consistently including all terms that are effectively of the same order in α_S, the PQCD prediction for an exclusive one-scale process is free of any residual dependence on the factorization scale at every order of the PQCD calculation. The unavoidable theoretical uncertainty of a particular order of the PQCD calculation remains only due to the renormalization procedure. Nevertheless, we comment on the problems one is left with when adopting such a procedure, especially in the case of multi-scale processes.

In Sect. 2 we introduce the ingredients of the standard hard-scattering picture on the example of the pion transition form factor, while in Sect. 3 the higher-order calculational procedure is outlined. The discussion of the factorization scale dependence is given in Sect. 4. Section 5 is devoted to concluding remarks.

2 Standard hard-scattering picture at higher-orders

For definiteness, notational simplicity, and clarity of presentation, we consider the high-energy behavior of the simplest exclusive quantity, the pion transition form factor $F_{\gamma\pi}(Q^2)$, defined in terms of the $\gamma^*(q,\mu)+\gamma(k,\nu)\to\pi(P)$ amplitude. For large momentum transfer $Q^2(=-q^2)$, the general factorization formula [1–4] for $F_{\gamma\pi}(Q^2)$ reads

$$F_{\gamma\pi}(Q^2) = \Phi^*(x,\mu_F^2) \otimes T_H(x,Q^2,\mu_F^2)\,. \qquad (1)$$

Here, $\Phi(x,\mu_F^2)$ is the pion distribution amplitude; $T_H(x,Q^2,\mu_F^2)$ is the hard-scattering amplitude; μ_F^2 is the factorization scale, and x denotes the pion constituent's momentum fraction, while $\otimes \equiv \int_0^1 \mathrm{d}x$.

The hard-scattering amplitude (HSA) T_H can be explicitly calculated in perturbation theory and represented as a series in the QCD running coupling constant $\alpha_S(\mu_R^2)$ by

$$T_H(x,Q^2,\mu_F^2) = T_H^{(0)}(x,Q^2) + \frac{\alpha_S(\mu_R^2)}{4\pi}\,T_H^{(1)}(x,Q^2,\mu_F^2) + \frac{\alpha_S^2(\mu_R^2)}{(4\pi)^2}\,T_H^{(2)}(x,Q^2,\mu_F^2,\mu_R^2)+\cdots\,, \qquad (2)$$

where μ_R^2 is the renormalization scale. The dependence of the coefficients of the expansion (2) on the scales μ_R^2 and μ_F^2 is of the form $\ln^n(\mu_R^2/Q^2)$ and $\ln^m(\mu_F^2/Q^2)$, respectively.

The pion distribution amplitude $\Phi(x,\mu_F^2)$, although intrinsically non-perturbative, satisfies the Brodsky–Lepage (BL) evolution equation

$$\mu_F^2\frac{\partial}{\partial\mu_F^2}\Phi(x,\mu_F^2) = V(x,u,\mu_F^2)\otimes\Phi(u,\mu_F^2)\,, \qquad (3)$$

where $V(x,u,\mu_F^2)$ is the perturbatively calculable evolution kernel

$$V(x,u,\mu_F^2) = \frac{\alpha_S(\mu_F^2)}{4\pi}V_1(x,u) + \frac{\alpha_S^2(\mu_F^2)}{(4\pi)^2}V_2(x,u)+\cdots \qquad (4)$$

The solution of (3) can be represented as

$$\Phi(x,\mu_F^2) = \Phi^{\mathrm{LO}}(x,\mu_F^2) + \frac{\alpha_S(\mu_F^2)}{4\pi}\;\Phi^{\mathrm{NLO}}(x,\mu_F^2)+\cdots\,, \qquad (5)$$

where Φ^{LO} and Φ^{NLO} denote the leading order (LO) and next-to-leading order (NLO) parts, respectively. When convoluting the finite-order results (2) and (5) according to (1), one is usually left with the residual dependence on both μ_R^2 and μ_F^2. The origin of the latter will be explained in the following.

3 Calculational procedure

In order to be able to examine the origin of the residual dependence on μ_F^2, we first reexamine the calculational procedure and the ingredients of the standard hard-scattering picture for $F_{\gamma\pi}(Q^2)$.

The HSA T_H is obtained by evaluating the $\gamma^*+\gamma\to q\bar{q}$ amplitude, which we denote by T. Owing to the fact that final-state quarks are taken to be massless and on-shell, the amplitude contains collinear singularities. Since T_H is a finite quantity by definition, collinear singularities have to be subtracted. Therefore, T factorizes as

$$T(u,Q^2) = T_H(x,Q^2,\mu_F^2)\otimes Z_{T,\mathrm{col}}(x,u;\mu_F^2)\,, \qquad (6)$$

with collinear singularities being subtracted at the scale μ_F^2 and absorbed into the constant $Z_{T,\mathrm{col}}$. The UV singularities are removed by the renormalization of the fields and by the coupling-constant renormalization at the (renormalization) scale μ_R^2.

The process-independent pion DA in a frame where $P^+ = P^0+P^3 = 1$, $P^- = P^0 - P^3 = 0$, and $P_\perp = 0$ is defined [4,12,13] as

$$\Phi(u) = \int\frac{\mathrm{d}z^-}{2\pi}\mathrm{e}^{\mathrm{i}(u-(1-u))z^-/2} \times\left\langle 0\left|\bar{\Psi}(-z)\,\frac{\gamma^+\gamma_5}{2\sqrt{2}}\,\Omega\,\Psi(z)\right|\pi\right\rangle_{(z^+=z_\perp=0)}\,, \qquad (7)$$

where $\Omega = \exp\left\{\mathrm{i}g\int_{-1}^1 \mathrm{d}s A^+(zs)z^-/2\right\}$ is a path-ordered factor making Φ gauge invariant. Owing to the light-cone singularity at $z^2=0$ [4,13] the matrix element in (7) is UV divergent. After regularization and renormalization at the scale $\tilde{\mu}_R^2$, z^2 is effectively smeared over a region of order $z^2 = -z_\perp^2 \sim 1/\tilde{\mu}_R^2$. As a result, a finite quantity, namely, the pion DA $\Phi(v,\tilde{\mu}_R^2)$, is obtained and corresponds to the pion wave function integrated over the pion intrinsic transverse momentum up to the scale $\tilde{\mu}_R^2$.

The pion DA as given in (7), with $|\pi\rangle$ being the physical pion state, cannot be determined by perturbation theory. If the meson state $|\pi\rangle$ is replaced by a $|q\bar{q};t\rangle$ state composed of a free (collinear, massless, and on-shell) quark and antiquark (carrying momenta tP and $(1-t)P$ and pseudoscalar meson quantum numbers), then the amplitude (7) becomes

$$\tilde{\phi}(u,t) = \int \frac{\mathrm{d}z^-}{2\pi} \mathrm{e}^{\mathrm{i}(u-(1-u))z^-/2} \times \left\langle 0 \left| \overline{\Psi}(-z) \frac{\gamma^+\gamma_5}{2\sqrt{2}} \Omega \Psi(z) \right| q\bar{q}; t \right\rangle . \quad (8)$$

Taking (8) into account, we can express (7) as

$$\Phi(u) = \tilde{\phi}(u,t) \otimes \langle q\bar{q}; t|\pi\rangle . \quad (9)$$

The distribution $\tilde{\phi}(u,t)$ can be treated perturbatively, which enables us to investigate the high-energy tail of the pion DA and its evolution. The $\tilde{\phi}(u,t)$ distribution is multiplicatively renormalizable owing to the multiplicative renormalizability of the composite operator $\overline{\Psi}(-z)\,\gamma^+\gamma_5\,\Omega\,\Psi(z)$. This means that the UV singularities that are not removed by the renormalization of the fields and by the coupling-constant renormalization factorize in the renormalization constant $Z_{\phi,\mathrm{ren}}$ at the (renormalization) scale $\tilde{\mu}_\mathrm{R}^2$. Apart from UV singularities, the matrix element in (8) contains also collinear singularities. Subtracting these singularities at the scale μ_0^2 and absorbing them in $Z_{\phi,\mathrm{col}}$, we can write (8) as

$$\tilde{\phi}(u,t) = Z_{\phi,\mathrm{ren}}(u,v;\tilde{\mu}_\mathrm{R}^2) \otimes \phi_V(v,s;\tilde{\mu}_\mathrm{R}^2,\mu_0^2) \otimes Z_{\phi,\mathrm{col}}(s,t;\mu_0^2) . \quad (10)$$

By combining (9) and (10), we obtain the distribution $\Phi(u)$ in the form

$$\Phi(u) = Z_{\phi,\mathrm{ren}}(u,v;\tilde{\mu}_\mathrm{R}^2) \otimes \Phi(v,\tilde{\mu}_\mathrm{R}^2) , \quad (11)$$

where

$$\Phi(v,\tilde{\mu}_\mathrm{R}^2) = \phi_V(v,s;\tilde{\mu}_\mathrm{R}^2,\mu_0^2) \otimes \Phi(s,\mu_0^2) . \quad (12)$$

Here,

$$\Phi(s,\mu_0^2) = Z_{\phi,\mathrm{col}}(s,t;\mu_0^2) \otimes \langle q\bar{q}; t|\pi\rangle \quad (13)$$

represents the non-perturbative input (containing collinear singularities and all effects of confinement and pion bound-state dynamics) determined at the scale μ_0^2, while $\phi_V(v,s;\tilde{\mu}_\mathrm{R}^2,\mu_0^2)$ governs the evolution of $\Phi(v,\mu_0^2)$ to the scale $\tilde{\mu}_\mathrm{R}^2$. By differentiating (11) with respect to $\tilde{\mu}_\mathrm{R}^2$, one obtains (3), with the evolution potential V given by

$$V(\tilde{\mu}_\mathrm{R}^2) = -Z^{-1}_{\phi,\mathrm{ren}}(\tilde{\mu}_\mathrm{R}^2) \left(\tilde{\mu}_\mathrm{R}^2 \frac{\partial}{\partial \tilde{\mu}_\mathrm{R}^2} Z_{\phi,\mathrm{ren}}(\tilde{\mu}_\mathrm{R}^2) \right) . \quad (14)$$

To simplify the expressions, the convolution ($\otimes$) is here, and where appropriate, replaced by the matrix multiplication in x–y space (the unit matrix is defined as $\mathbb{1} = \delta(x-y)$), while the x, y variables are suppressed.

By convoluting the amplitudes $T(u,Q^2)$ and $\Phi(u)$, (6) and (11), respectively, in analogy with [14,4] we obtain the pion transition form factor $F_{\gamma\pi}(Q^2)$:

$$F_{\gamma\pi}(Q^2) = \Phi^\dagger(u) \otimes T(u,Q^2) . \quad (15)$$

Now, in order that the factorization holds, $\tilde{\mu}_\mathrm{R}^2$ has to coincide with μ_F^2 and by making use of the fact that

$$Z_{T,\mathrm{col}}(x,u;\mu_\mathrm{F}^2) \otimes Z_{\phi,\mathrm{ren}}(u,v;\mu_\mathrm{F}^2) = \delta(x-v) , \quad (16)$$

the divergences of $T(u,Q^2)$ and $\Phi(u)$ in (15) cancel (this has been explicitly shown in [15] up to n_f-proportional terms of $O(\alpha_s^2)$) and we are left with the finite perturbative expression for the pion transition form factor (1).

We note here that the same factorization (and renormalization) scheme is employed in the hard-scattering and DA part, i.e., in (6) and (11), respectively. Furthermore, as pointed out in [16], the evolution equation as defined by (3) and (4) corresponds to the simplified scheme fixed by the preference that the distribution amplitude should have no dependence on the renormalization scale[1].

It is worth pointing out that the scale μ_F^2 representing the boundary between the low- and high-energy parts in (1) plays the role of the separation scale for collinear singularities in $T(u,Q^2)$, on one hand, and of the renormalization scale for UV singularities appearing in the perturbatively calculable part of the distribution amplitude $\Phi(u)$, on the other hand.

The calculational procedure explained above is illustrated in Fig. 1.

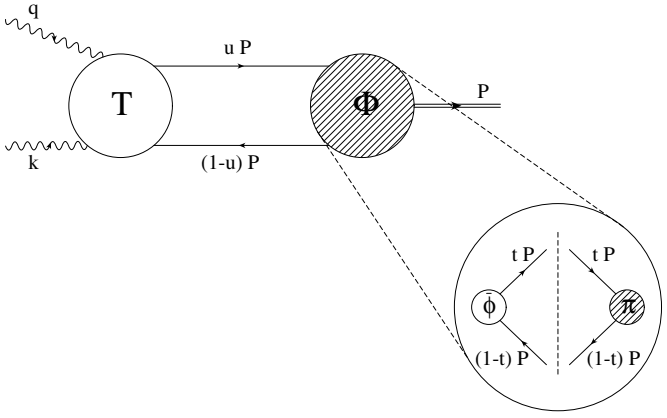

Fig. 1. Pictorial representation of the calculational ingredients of the pion transition form factor. T represents the perturbatively calculable $\gamma^* + \gamma \to q\bar{q}$ hard-scattering amplitude, while Φ denotes the (unrenormalized) pion distribution amplitude given by (7), which can be expressed, as in (9), in terms of the perturbatively calculable part $\tilde{\phi}$ (8) and a perturbatively incalculable part

[1] Note that, in general, such a residual dependence appears along with the evolution kernel depending on two scales:

$$V(x,u,\mu_\mathrm{F}^2) = \frac{\alpha_s(\mu_\mathrm{R}^2)}{4\pi} V_1(x,u) + \frac{\alpha_s^2(\mu_\mathrm{R}^2)}{(4\pi)^2} \left(V_2(x,u) - \beta_0 V_1(x,u) \ln\left(\frac{\mu_\mathrm{R}^2}{\mu_\mathrm{F}^2}\right)\right) + O\left(\alpha_s^3\right) .$$

Here μ_R^2 corresponds to the scale of the coupling constant, while $\tilde{\mu}_\mathrm{R}^2 = \mu_\mathrm{F}^2$ denotes the scale at which remaining UV divergences, due to the renormalization of the composite operator, factorize.

4 Factorization scale dependence

We next turn to the discussion of the $\mu_{\rm F}^2$ dependence of the pion transition form factor defined as in (1).

Concerning the pion distribution amplitude $\Phi(x,\mu_{\rm F}^2)$, its dependence on $\mu_{\rm F}^2$ is specified by the evolution equation (3) and, as can be seen from (12), this dependence is completely contained in the evolutional part ϕ_V. By calculating the perturbatively obtainable amplitude $\tilde{\phi}$ [15] directly from (8), the result obtained for ϕ_V can be organized as

$$\phi_V(\mu_{\rm F}^2,\mu_0^2) = \phi_V^{\rm LO}(\mu_{\rm F}^2,\mu_0^2) + \frac{\alpha_{\rm S}(\mu_{\rm F}^2)}{4\pi}\phi_V^{\rm NLO}(\mu_{\rm F}^2,\mu_0^2) + \cdots , \tag{17}$$

where

$$\phi_V^{\rm LO}(\mu_{\rm F}^2,\mu_0^2) = \mathbb{1} + \frac{\alpha_{\rm S}(\mu_{\rm F}^2)}{4\pi}\ln\frac{\mu_{\rm F}^2}{\mu_0^2}\,V_1 + \frac{\alpha_{\rm S}^2(\mu_{\rm F}^2)}{(4\pi)^2}\ln^2\frac{\mu_{\rm F}^2}{\mu_0^2}\,\frac{1}{2}\,(V_1^2+\beta_0\,V_1) + \cdots , \tag{18a}$$

$$\phi_V^{\rm NLO}(\mu_{\rm F}^2,\mu_0^2) = \frac{\alpha_{\rm S}(\mu_{\rm F}^2)}{4\pi}\ln\frac{\mu_{\rm F}^2}{\mu_0^2}\,V_2 + \cdots , \tag{18b}$$

and the functions V_n represent the n-loop evolutional kernels appearing in (3). The terms explicitly given in (18) correspond to the results of the two-loop calculation [15]. In writing (18), use has been made of

$$\frac{\alpha_{\rm S}(\mu_{\rm F}^2)}{4\pi}\ln\frac{\mu_{\rm F}^2}{\mu_0^2} \approx \frac{1}{\beta_0}\left(1-\frac{\alpha_{\rm S}(\mu_{\rm F}^2)}{\alpha_{\rm S}(\mu_0^2)}\right) = O(\alpha_{\rm S}^0)\,.$$

On the other hand, the complete LO and NLO behavior of $\phi_V(v,s;\mu_{\rm F}^2,\mu_0^2)$ and, consequently, of $\Phi(v,\mu_{\rm F}^2)$ can be determined by solving the evolution equation (3) or equivalently

$$\mu_{\rm F}^2\frac{\partial}{\partial\mu_{\rm F}^2}\phi_V(v,s,\mu_{\rm F}^2,\mu_0^2) = V(v,s',\mu_{\rm F}^2)\otimes\phi_V(s',s,\mu_{\rm F}^2,\mu_0^2)\,. \tag{19}$$

The LO result is of the form

$$\phi_V^{\rm LO}(v,s;\mu_{\rm F}^2,\mu_0^2) = \sum_{n=0}^{\infty}{}' \frac{v(1-v)}{N_n}\,C_n^{3/2}(2v-1)C_n^{3/2}(2s-1) \times\left(\frac{\alpha_{\rm S}(\mu_{\rm F}^2)}{\alpha_{\rm S}(\mu_0^2)}\right)^{-\gamma_n^{(0)}/\beta_0}, \tag{20}$$

where $N_n=(n+1)(n+2)/(4(2n+3))$, while $C_n^{3/2}(2x-1)$ are the Gegenbauer polynomials (the eigenfunctions of the LO kernel V_1 with the corresponding eigenvalues $\gamma_n^{(0)}$ [11]). The complete LO prediction given above represents the summation of all $(\alpha_{\rm S}\ln\mu_{\rm F}^2/\mu_0^2)^n$ terms from (18a). The complete formal solution of the NLO evolution equation was obtained in [17] by using conformal constraints and the form of $\phi_V^{\rm NLO}$ (corresponding to the resummation of (18b)) can be extracted from the results listed in [11].

It is important to realize that the method employed above to study the $\mu_{\rm F}^2$ behavior of ϕ_V can be used to examine the dependence of the hard-scattering amplitude $T_{\rm H}(x,Q^2,\mu_{\rm F}^2)$ on the scale $\mu_{\rm F}^2$, as well.

By differentiating (1) with respect to $\mu_{\rm F}^2$ and by taking into account (3), one finds that the hard-scattering amplitude satisfies the evolution equation

$$\mu_{\rm F}^2\frac{\partial}{\partial\mu_{\rm F}^2}T_{\rm H}(x,Q^2,\mu_{\rm F}^2) = -T_{\rm H}(y,Q^2,\mu_{\rm F}^2)\otimes V(y,x;\mu_{\rm F}^2)\,. \tag{21}$$

This equation[2] is analogous to the DA evolution equation (3). Similarly to the above discussed solution of the DA evolution equation, the finite-order solution of (21) would contain the complete dependence on $\mu_{\rm F}^2$, to given order in $\alpha_{\rm S}$, in contrast to the expansion (2) truncated at the same order and containing unresummed logs. Let us note that the explicit expressions for the hard-scattering amplitude $T_{\rm H}(x,Q^2,\mu_{\rm F}^2)$ in the form (2), evaluated up to n_f-proportional NNLO terms, are given in [15].

The $\mu_{\rm F}^2$ dependence of $T_{\rm H}(x,Q^2,\mu_{\rm F}^2)$ can be, similarly to (12), factorized in the function $\phi_V(y,x,Q^2,\mu_{\rm F}^2)$ as follows:

$$T_{\rm H}(x,Q^2,\mu_{\rm F}^2) = T_{\rm H}(y,Q^2,\mu_{\rm F}^2=Q^2)\otimes\ \phi_V(y,x,Q^2,\mu_{\rm F}^2)\,. \tag{22}$$

Using (19) one can show by partial integration that (22) indeed represents the solution of the evolution equation (21).

When calculating to finite order in $\alpha_{\rm S}$, it seems not quite consistent to adopt the procedure, often encountered in the literature, in which the $\Phi(x,\mu_{\rm F}^2)$ distribution obtained by solving the evolution equation (3) is convoluted with $T_{\rm H}(x,Q^2,\mu_{\rm F}^2)$ obtained by the truncation of the expansion (2). In the latter case, only the partial dependence on $\mu_{\rm F}^2$ is included (logs are not resummed), in contrast to the former case and hence the residual dependence on the factorization scale $\mu_{\rm F}^2$ enters.

The proper procedure would be to convolute Φ (12) and $T_{\rm H}$ (22) in terms of the same function ϕ_V, where ϕ_V can be given by (18) with unresummed logs or can represent the solution of (19), i.e., the resummed result. In both cases the $\mu_{\rm F}^2$ dependence of Φ and $T_{\rm H}$ completely cancels out and there is no residual dependence on $\mu_{\rm F}^2$. One usually uses the resummed form of ϕ_V in DA Φ, i.e, Φ is taken as a solution of the evolution equation, and therefore this procedure should be applied for $T_{\rm H}$ as well.

Substituting (12) and (22) in (1), we obtain

$$F_{\gamma\pi}(Q^2) = T_{\rm H}(y,Q^2,Q^2)\otimes\phi_V(y,s,Q^2,\mu_0^2)\otimes\Phi^*(s,\mu_0^2)\,, \tag{23}$$

[2] Equation (21) can be also obtained by combining (6) with (14) and (16).

where

$$\phi_V(y,x,Q^2,\mu_{\rm F}^2)\otimes\phi_V(x,s,\mu_{\rm F}^2,\mu_0^2) = \phi_V(y,s,Q^2,\mu_0^2) \quad (24)$$

has been taken into account. It is important to realize that the expression (24) is valid at every order of a PQCD calculation[3], and hence even the finite-order prediction for $F_{\gamma\pi}(Q^2)$ does not depend on the choice of the $\mu_{\rm F}^2$ scale[4].

Hence, the expression (24) represents the resummation of the $\ln(Q^2/\mu_0^2)$ logarithms over the intermediate $\mu_{\rm F}^2$ scale, performed in such a way that both the logarithms $\ln(\mu_{\rm F}^2/\mu_0^2)$ originating from the perturbative part of the DA and the $\ln(Q^2/\mu_{\rm F}^2)$ logarithms from the hard-scattering part are resummed. The effect in the final prediction, at every order, is the same as if we had performed the complete renormalization-group resummation of the $\ln(Q^2/\mu_0^2)$ logarithms.

Although by using the explicit results for (2) and the evolution equation solution for ϕ_V it is straightforward to employ (22) and obtain $T_{\rm H}$ with resummed $\ln(\mu_{\rm F}^2/Q^2)$ logs, it is much easier that the complete resummation is performed in the distribution amplitude. Hence, by adopting the common choice $\mu_{\rm F}^2 = Q^2$, we avoid the need for the resummation of the $\ln(Q^2/\mu_{\rm F}^2)$ logarithms in the hard-scattering part, making the calculation simpler and hence, for practical purposes, the preferable form of $F_{\gamma\pi}(Q^2)$ is given by

$$F_{\gamma\pi}(Q^2) = T_{\rm H}(x,Q^2,Q^2)\otimes\Phi^*(x,Q^2)\,. \quad (25)$$

We stress here that in this approach, in which the consistent treatment of Φ and $T_{\rm H}$ dependence on $\mu_{\rm F}^2$ is required, any other choice of $\mu_{\rm F}^2$ would lead to the same result; only the calculation would be more involved.

[3] Equation (24) can be easily checked to the NLO order [15] by using the LO result (20) and the NLO results of [17].

[4] Let us, following (4) and (17), define the finite-order quantities

$$\phi_V^{(n)}(\mu_{\rm F}^2,\mu_0^2) = \phi_V^{\rm LO}(\mu_{\rm F}^2,\mu_0^2)+\cdots+\frac{\alpha_{\rm S}^n(\mu_{\rm F}^2)}{(4\pi)^n}\phi_V^{\rm N\cdots NLO}(\mu_{\rm F}^2,\mu_0^2)\,,$$

and

$$V^{(n)}(\mu_{\rm F}^2) = \frac{\alpha_{\rm S}(\mu_{\rm F}^2)}{4\pi}V_1+\cdots+\frac{\alpha_{\rm S}^{n+1}(\mu_{\rm F}^2)}{(4\pi)^{n+1}}V_n$$

(here $n=0,\ldots$). The functions $\phi_V^{(n)}(Q^2,\mu_{\rm F}^2)$ and $\phi_V^{(n)}(\mu_{\rm F}^2,\mu_0^2)$ represent the solutions of the evolutional equations

$$\mu_{\rm F}^2\frac{\partial}{\partial\mu_{\rm F}^2}\phi_V^{(n)}(\mu_{\rm F}^2,\mu_0^2) = V^{(n)}(\mu_{\rm F}^2)\otimes\phi_V^{(n)}(\mu_{\rm F}^2,\mu_0^2)\,,$$

$$\mu_{\rm F}^2\frac{\partial}{\partial\mu_{\rm F}^2}\phi_V^{(n)}(Q^2,\mu_{\rm F}^2) = -\phi_V^{(n)}(Q^2,\mu_{\rm F}^2)\otimes V^{(n)}(\mu_{\rm F}^2)\,.$$

It is now easy to prove that the convolution

$$\phi_V^{(n)}(Q^2,\mu_{\rm F}^2)\otimes\phi_V^{(n)}(\mu_{\rm F}^2,\mu_0^2)$$

indeed does not depend on $\mu_{\rm F}^2$.

5 Concluding remarks

We have sketched the higher-order PQCD calculational procedure for the hard exclusive quantities on the example of the pion transition form factor $F_{\gamma\pi}$.

Furthermore, we have argued that the $F_{\gamma\pi}$ prediction (1) is independent of the factorization scale $\mu_{\rm F}^2$ at every order in $\alpha_{\rm S}$, when both the hard-scattering part $T_{\rm H}$ and the distribution amplitude Φ are consistently treated regarding the $\mu_{\rm F}^2$ dependence, i.e., in both quantities the $\ln\mu_{\rm F}^2$ logarithms are resummed or in both quantities they are not resummed. The $\mu_{\rm F}^2$ dependence of Φ then exactly cancels the $\mu_{\rm F}^2$ dependence of $T_{\rm H}$, and the choice of the factorization scale is therefore non-essential, and the predictions obtained by using any choice of $\mu_{\rm F}^2$ are equal to the results obtained using for practical purposes the simplest intermediate choice $\mu_{\rm F}^2 = Q^2$, where Q^2 represents the characteristic scale of the process.

The true expansion parameter left is $\alpha_{\rm S}(\mu_{\rm R}^2)$, with $\mu_{\rm R}^2$ representing the renormalization scale of the complete perturbatively calculable part of the pion transition form factor (23), i.e., of

$$T_{\rm H}(s,Q^2,\mu_0^2) = T_{\rm H}(y,Q^2,Q^2)\otimes\phi_V(y,s,Q^2,\mu_0^2)\,. \quad (26)$$

Therefore, although $F_{\gamma\pi}(Q^2)$ depends exclusively on the characteristic scale of the process Q^2, we are left with the residual dependence on the $\mu_{\rm R}^2$ scale, when calculating to finite order. The intermediate scale at which the short- and long-distance dynamics separate, the factorization scale, disappears from the final prediction at every order in $\alpha_{\rm S}$ and therefore does not introduce any theoretical uncertainty into the PQCD calculation for exclusive processes.

The above discussed calculational prescription for the factorization scale independent calculation is also upheld for other PQCD exclusive one-scale processes. However, in the case of exclusive processes which involve more than one typical scale the treatment of the factorization scale dependence is more involved. The subtlety in the preceding approach lies in the fact that we have traded one dependence on $\mu_{\rm F}$ for another one. Namely, the choice we have made there is that we resum ln terms up to the relevant scale of the process Q^2. Although this might seem reasonable for the one external scale processes, such as the one which define $F_{\gamma\pi}$ ($\gamma\gamma^*\to\pi$) or the pion electromagnetic form factor, in processes with two scales, for example in the one in which the general pion transition form factor $F_{\gamma^*\pi}$ ($\gamma^*\gamma^*\to\pi$) appears, one immediately encounters the ambiguity of how to choose the relevant scale up to which the logs will be resummed. The existence of such ambiguities seems to be an unavoidable artefact of the PQCD calculation.

Acknowledgements. We would like to thank P. Kroll, D. Müller and A. Radyushkin for useful comments. One of us (B.M.) acknowledges the support by the Alexander von Humboldt Foundation and the hospitality of the theory groups at the Institut für Physik, Universität Mainz and Institut für Theoretische Physik, Universität Würzburg. This work was supported by the

Ministry of Science and Technology of the Republic of Croatia under Contract No. 00980102.

References

1. G.P. Lepage, S.J. Brodsky, Phys. Lett. B **87**, 359 (1979); Phys. Rev. Lett. **43**, 545 (1979)
2. A.V. Efremov, A.V. Radyushkin, Theor. Math. Phys. **42**, 97 (1980); Phys. Lett. B **94**, 245 (1980)
3. A. Duncan, A.H. Mueller, Phys. Lett. B **90**, 159 (1980); Phys. Rev. D **21**, 1636 (1980)
4. G.P. Lepage, S.J. Brodsky, Phys. Rev. D **22**, 2157 (1980)
5. G. Grunberg, Phys. Rev. D **29**, 2315 (1984)
6. P.M. Stevenson, Nucl. Phys. B **231**, 65 (1984)
7. S.J. Brodsky, G.P. Lepage, P.B. Mackenzie, Phys. Rev. D **28**, 228 (1983)
8. S.J. Brodsky, C.-R. Ji, A. Pang, D.G. Robertson, Phys. Rev. D **57**, 245 (1998)
9. H. Nakkagawa, A. Niegawa, Phys. Lett. B **119**, 415 (1982); P.M. Stevenson, H.D. Politzer, Nucl. Phys. B **277**, 758 (1986); C.J. Maxwell, Nucl. Phys. B **577**, 209 (2000)
10. F.M. Dittes, A.V. Radyushkin, Yad. Fiz. **34**, 529 (1981) [Sov. J. Nucl. Phys. **34**, 293 (1981)]
11. B. Melić, B. Nižić, K. Passek, Phys. Rev. D **60**, 074004 (1999)
12. G.R. Katz, Phys. Rev. D **31**, 652 (1985); Ph.D. Thesis (1986)
13. S.J. Brodsky, P. Damgaard, Y. Frishman, G.P. Lepage, Phys. Rev. D **33**, 1881 (1986)
14. G. Curci, W. Furmanski, R. Petronzio, Nucl. Phys. B **175**, 27 (1980)
15. B. Melić, B. Nižić, K. Passek, Phys. Rev. D **65**, 053020 (2001)
16. B. Melić, K. Passek-Kumerički, D. Müller, Phys. Rev. D **68**, 014013 (2003)
17. D. Müller, Phys. Rev. D **49**, 2525 (1994); D **51**, 3855 (1995)